C2 June 2011 (MA)

Q1a) $f(1) = 2(1) - 7(1) - 5(1) + 4 = 6 - 12 = \boxed{-6}$

b) $f(-1) = 2(-1) - 7 + 5 + 4 = 9 - 9 = 0$ //

$\therefore (x+1)$ is a factor.

c) $f(x) =$

$$\begin{array}{r|l}
 & 2x^2 - 9x + 4 \\
\hline
x+1 & 2x^3 - 7x^2 - 5x + 4 \\
 & 2x^3 + 2x^2 \\
\cline{2-2}
 & 0 \quad -9x^2 - 5x \\
 & \qquad -9x^2 - 9x \\
\cline{2-2}
 & \qquad 0 \quad +4x + 4 \\
 & \qquad\qquad 4x + 4 \\
\cline{2-2}
 & \qquad\qquad 0 \quad 0
\end{array}$$

$\therefore f(x) = (x+1)(2x^2 - 9x + 4)$

$2x^2 - 9x + 4 = (2x - 1)(x - 4)$

$\therefore f(x) = \boxed{(x+1)(2x-1)(x-4)}$

Q2a) $(3+bx)^5 \simeq (3)^5 + \binom{5}{1}(3)^4(bx)^1 + \binom{5}{2}(3)^3(bx)^2$

$\simeq 243 + 405bx + 270b^2x^2$

b) $270b^2 = 2(405b)$

$270b^2 - 810b = 0$

$b(270b - 810) = 0$

$b \neq 0 \quad \therefore 270b - 810 = 0$

$b = \dfrac{810}{270} = \boxed{3}$

3a) $5^x = 10$

$\log(5^x) = \log(10)$

$x \log 5 = \log 10$

$x = \dfrac{\log 10}{\log 5} = \boxed{1.43}$

b) $\log_3(x-2) = -1$

$3^{-1} = x - 2 = \frac{1}{3}$

$x = 2 + \frac{1}{3}$

$\boxed{x = \frac{7}{3}}$

4a) $x^2 + 4x + y^2 - 2y - 11 = 0$

$(x+2)^2 - 4 + (y-1)^2 - 1 - 11 = 0$

$(x+2)^2 + (y-1)^2 = 16$

$\therefore$ centre $(-2, 1)$

b) radius $= \sqrt{16} = \boxed{4}$

c) $x = 0$: $(2)^2 + (y-1)^2 = 16$

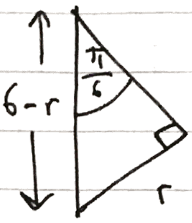
$(y-1)^2 = 12$

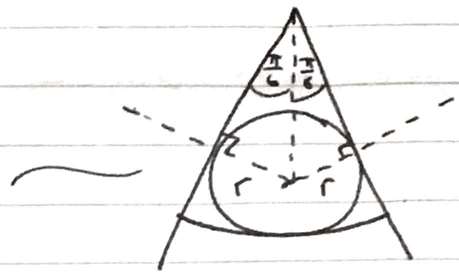
$y - 1 = \pm\sqrt{12}$

$y = 1 \pm 2\sqrt{3}$ $\therefore$ $\boxed{(0, 1+2\sqrt{3})}$ $\boxed{(0, 1-2\sqrt{3})}$

5a) Area $= \frac{1}{2}r^2\theta = \frac{1}{2}(6^2)\left(\frac{\pi}{3}\right) = \boxed{6\pi}$

b) $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$





$\therefore \sin\frac{\pi}{6} = \frac{r}{6-r}$

$\frac{1}{2} = \frac{r}{6-r} \Rightarrow 3 - \frac{r}{2} = r$

$\frac{3r}{2} = 3 \quad \therefore \boxed{r = 2}$

c) Area $= 6\pi - \pi(2^2) = \boxed{2\pi}$

6a) $a \quad ar \quad ar^2$

$\left.\begin{array}{l} ar = 192 \\ ar^2 = 144 \end{array}\right\} \frac{ar^2}{ar} = \frac{144}{192} = \boxed{\frac{3}{4} = r}$

b) $a = \frac{192}{r} = \frac{192}{\frac{3}{4}} = \boxed{256}$

c) $S_\infty = \frac{a}{1-r} = \frac{256}{1-\frac{3}{4}} = \boxed{1024}$

d) $S_N = \frac{a(1-r^n)}{1-r} = \frac{256(1-(\frac{3}{4})^n)}{1-\frac{3}{4}} > 1000$

$256(1-\frac{3}{4}^n) > 250$

$1 - \frac{3}{4}^n > \frac{250}{256}$

$\therefore \left(\frac{3}{4}\right)^n < 1 - \frac{250}{256}$

$\left(\frac{3}{4}\right)^n < \frac{3}{128}$

$\log\left(\frac{3}{4}^n\right) < \log\left(\frac{3}{128}\right)$

$n \log\left(\frac{3}{4}\right) < \log\left(\frac{3}{128}\right)$

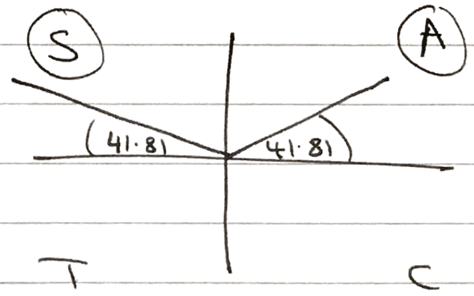
$n > \frac{\log\frac{3}{128}}{\log\left(\frac{3}{4}\right)} \quad \therefore \; n > 13.047\ldots$

$\therefore \boxed{n_{min} = 14}$

signs change when $\div$ by a negative number. ($\log\frac{3}{4} < 0$).

7a) $\sin(x+45) = \frac{2}{3} \quad \therefore x + 45 = \sin^{-1}\left(\frac{2}{3}\right) = 41.81°$

solving in: $45 \leq x+45 < 405$



$x + 45° = (180 - 41.81),$
$(360 + 41.81)$

$x + 45 = 138.19, 401.81$

$x = \boxed{93.2°}, \boxed{356.8°}$

b) $0 \leq x < 2\pi$

$$2\sin^2 x + 2 = 7\cos x$$
$$2 - 2\cos^2 x + 2 = 7\cos x$$
$$2\cos^2 x + 7\cos x - 4 = 0$$
$$(2\cos x - 1)(\cos x + 4) = 0$$

$2\cos x - 1 = 0$

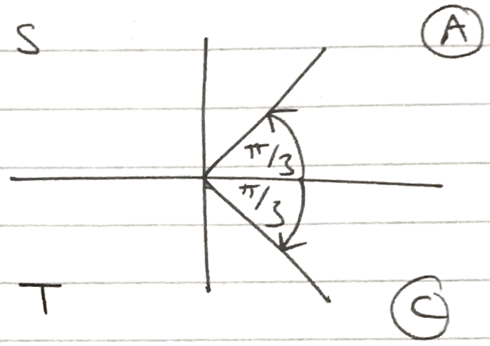
$\cos x = \frac{1}{2}$

$x = \cos^{-1}\left(\frac{1}{2}\right) = 60°. = \frac{\pi}{3}$

$\cos x + 4 = 0$

$\cos x = -4$ × reject.

[NO VALID SOLUTIONS]



$x = \frac{\pi}{3}, \; 2\pi - \frac{\pi}{3}$

$$\boxed{x = \frac{\pi}{3}, \; \frac{5\pi}{3}}$$

8a) $V = (x) \times (2x) \times (y) = 81$

height (unknown)

$\therefore \; 2x^2 y = 81$

$\therefore \; y = \frac{81}{2x^2}$

$L = 2x + 2x + 2x + 2x + 4(x) + 4(y)$

$L = 12x + 4y = 12x + 4\left(\frac{81}{2x^2}\right)$

$\therefore \; L = 12x + \frac{162}{x^2}$

b) $L = 12x + 162x^{-2}$

$$\frac{dL}{dx} = 12 - 324x^{-3} = 0$$

$$12 = \frac{324}{x^3}$$

$$x^3 = 27 \quad \left(= \frac{324}{12}\right)$$

$$\therefore \overline{x = 3} \mathbin{/\!/} \rightarrow L = 12(3) + \frac{162}{3^2} = \boxed{54} \text{ cm.}$$

c) $\frac{d^2L}{dx^2} = 972x^{-4} > 0$ when $x = 3$ //

(and for any value of x since $x > 0$)

$\therefore$ Value of L found is minimum.

9a) $x + 4 = -x^2 + 2x + 24$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = 5, \; x = -4$ //

$y = 5 + 4 = 9 \qquad y = -4 + 4 = 0$

$\therefore A(-4, 0)$ and $B(5, 9)$

b) $$R = \int_{-4}^{5} [y_2 - y_1]\,dx = \int_{-4}^{5} [-x^2 + 2x + 24 - x - 4]\,dx$$

$$= \int_{-4}^{5} [-x^2 + x + 20]\,dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x\right]_{-4}^{5}$$

$$= \left[-\frac{125}{3} + \frac{25}{2} + 100\right] - \left[\frac{64}{3} + 8 - 80\right]$$

$$= \frac{425}{6} + \frac{152}{3} = \boxed{121.5}$$